Multiple Beacon Based Robust Cooperative Spectrum Sensing in MIMO Cognitive Radio Networks under CSI Uncertainty

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***Abstract*—This paper presents multiple beacon vectors based robust detection schemes for cooperative spectrum sensing (CSS) in multiple-input multiple-output (MIMO) cognitive radio (CR) networks under channel state information (CSI) uncertainty. The inaccuracies in the estimate of the CSI are modeled as the standard ellipsoidal uncertainty set. We develop a multiple beacon vector based linear discriminant framework to obtain robust detectors for the problem of primary user detection in MIMO cognitive radio networks under ellipsoidal CSI uncertainty. Next, we employ this framework to develop signaling scheme based application specific detectors, namely the antipodal signaling based robust detector and on-off signaling based robust detector, along with their closed form expressions. Further, for the two signalling schemes, we even present allied detectors that are advantageous under low signal-to-noise ratio (SNR) conditions. Simulation results demonstrate a superior detection performance of the proposed robust detection schemes in comparison to the uncertainty agnostic matched filter detector for CSS in MIMO cognitive radio networks.**

I. Introduction

The continuing increase in the demand for higher data rates in emerging broadband wireless technologies coupled with the underutilization of spectrum resources arising due to the static spectrum allocation to the licensed/ primary users (PU) has inspired the introduction of cognitive radio systems. Cognitive radio networks [1], [2] permits the unlicensed/ secondary users that can opportunistically detect and reuse the unused licensed band of the primary users, thereby increasing effective spectrum utilization. Hence, the essential task for the cognitive radio users is to reliably detect the presence of vacant spectral bands, known as *spectrum sensing*, in order to reuse the unused spectrum without causing harmful interference to the primary communication.

In this regard, several single/ multiple secondary user based spectrum sensing techniques have been proposed [3]–[5]. Works, such as [4], [6], [7], illustrates a superior detection performance of multiple secondary user based cooperative spectrum sensing schemes over single secondary user based spectrum sensing schemes. However, cooperative spectrum sensing schemes are inherently channel state information (CSI) dependent. Moreover, obtaining accurate CSI in such multiuser wireless scenarios is non realistic due to the limited feedback and time varying nature of the wireless channel. Works, such as [8], [9], show drastic deterioration in the detection performance of such schemes with the increase in the uncertainty of the estimated CSI statistics. Hence, it is essential to develop detection schemes that are robust against CSI uncertainties.

In recent years, robust optimization in multiple-input multiple-output (MIMO) systems under CSI uncertainties have been actively researched. These are usually addressed as either stochastic approach or the worst-case approach. Stochastic approaches model the CSI uncertainty as Gaussian random variable [10], [11] in order to provide robustness in the statistical sense. Alternatively, worst-case approaches employ bounded CSI uncertainty model [12], [13] to obtain robustness under worst-case channel conditions. In contrast, such optimization framework to obtain robust decision rule in cognitive radio networks, under CSI uncertainty, is not much explored. Some recent works in [14], [15] consider stochastic approach and models CSI uncertainty as Gaussian random variable to present various optimal likelihood ratio test (LRT) based robust decision rules for spectrum sensing in MIMO cognitive radio scenario under CSI uncertainty. Another work [16] considers bounded CSI uncertainty to obtain detection schemes that are robust even under worst case CSI conditions for cooperative spectrum sensing in cognitive radio networks.

This paper proposes multiple beacon vectors based robust decision rules for cooperative spectrum sensing in MIMO cognitive radio networks, based on the linear classifier approach proposed in [16]. We first model the inaccuracy of the channel coefficient as the generalized ellipsoidal uncertainty set centered at the nominal estimate of the channel coefficients. We employ optimal linear discriminant framework to formulate multiple beacon vectors based cooperative spectrum sensing problem, considering ellipsoidal CSI uncertainty, as a tractable convex optimization problem. Then we even obtain multiple beacon based robust detectors for both antipodal and on-off signaling schemes along with their closed form expressions for realistic MIMO cognitive radio scenarios. Next, we present a framework for multiple beacon vector based allied robust detection schemes, namely the relax detector and the tradeoff detector, that are useful in deep fade and low signal-to-noise ratio (SNR) scenarios. The results in the simulation section

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validates an improved detection performance of the proposed robust detectors over the conventional uncertainty agnostic matched filter detector.

The manuscript denote scalars by lowercase letters, vectors by lowercase bold letters and matrices by uppercase bold letters where the matrix $\mathbf{I}_N$ is the identity matrix of dimension $N \times N$. The operators $\|\cdot\|_2$, $\otimes$ and vec$(\cdot)$ denotes L2-norm, kronecker product and the vector operation, respectively. The paper is organised as follows. Section II describes the system model for the problem of primary user detection in MIMO cognitive radio networks. Section III formulates the framework to compute robust detectors along with the allied detectors that consider uncertainty in the obtained CSI estimate. Simulation section IV validates the detection performance of the proposed schemes and finally we conclude in section V.

II. System Model

Consider a cognitive radio network with a centrally co-ordinated fusion center and N secondary users each having N_r receive antennas. The primary users communicate through the primary user base station with N_t transmit antennas. Hence, the secondary users sense the primary user base station signal in order to detect the presence of spectral hole. The received signal vector $\mathbf{y}_i(n) \in \mathbb{C}^{N_r \times 1}$ at the ith secondary user corresponding to the primary user base-station broadcast beacon signal $\mathbf{u}(n) \in \mathbb{C}^{Nt \times 1}, 1 \leq n \leq L$ is obtained as,

$$\mathbf{y}_i(n) = \mathbf{H}_i \mathbf{u}(n) + \mathbf{w}_i(n),$$

where the vector $\mathbf{w}_i(n) \in \mathbb{C}^{Nr \times 1}$ is zero mean additive white Gaussian noise (AWGN) at the ith secondary user at instant n with covariance matrix $E\{\mathbf{w}_i(n)\mathbf{w}_i(n)^H\} = \sigma^2 \mathbf{I}_{N_r}$. The channel state information (CSI) matrix $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_t}$ denote channel coefficient matrix between the primary user base station and the ith secondary user. Each element $h_{i_{r,t}}$ of the ith channel coefficient matrix $\mathbf{H}_i$ represents the flat fading gain between tth transmit antenna and rth receive antenna. Consider a cognitive radio scenario in which the primary user base station broadcasts L beacon signal vectors prior to the primary transmission indicating the presence and absence of the primary signal. Therefore, the concatenated signal $\mathbf{Y}_i$ corresponding to the L broadcast beacon vectors $\mathbf{u}(1), \mathbf{u}(2), \ldots \mathbf{u}(L)$, i.e. the beacon matrix $\mathbf{U} = [\mathbf{u}(1), \mathbf{u}(2), \ldots, \mathbf{u}(L)] \in \mathbb{C}^{Nt \times L}$, at ith secondary user can be equivalently written as,

$$\mathbf{Y}_i = \mathbf{H}_i \mathbf{U} + \mathbf{W}_i,$$

where matrix $\mathbf{Y}_i = [\mathbf{y}_i(1), \mathbf{y}_i(2), \ldots, \mathbf{y}_i(L)] \in \mathbb{C}^{Nr \times L}$ and matrix $\mathbf{W}_i = [\mathbf{w}_i(1), \mathbf{w}_i(2), \ldots, \mathbf{w}_i(L)] \in \mathbb{C}^{Nr \times L}$. Each secondary user senses the primary user broadcast beacon matrix $\mathbf{U}$ and send their observations to the centrally coordinated fusion center. Fusion center jointly processes the collected observations from N cooperating secondary users towards the detection of the spectral hole, i.e. absence of the licensed primary user. Hence the concatenated fusion center signal $\mathbf{Y} = [\mathbf{Y}_1^T, \ldots, \mathbf{Y}_i^T, \ldots, \mathbf{Y}_N^T]^T \in \mathbb{C}^{NNr \times L}$ corresponding to N secondary users can be equivalently written as,

$$\mathbf{Y} = \mathbf{H}\mathbf{U} + \mathbf{W} \tag{1}$$

where the concatenated channel matrix is defined as $\mathbf{H} = [\mathbf{H}_1^T, \ldots, \mathbf{H}_i^T, \ldots, \mathbf{H}_N^T]^T \in \mathbb{C}^{NN_r \times N_t}$ and similarly the concatenated noise matrix $\mathbf{W} = [\mathbf{W}_1^T, \ldots, \mathbf{W}_i^T, \ldots, \mathbf{W}_N^T]^T \in \mathbb{C}^{NNr \times L}$. Consider the transmission of a generalized beacon matrix $\mathbf{U} = \{\mathbf{P}_0, \mathbf{P}_1\} \in \mathbb{C}^{Nt \times L}$, prior to the primary transmission, indicating the absence and presence of the primary user signal. The beacon matrices $\mathbf{P}_0, \mathbf{P}_1$ can employ either antipodal or on-off signaling schemes. Let the vector $\mathbf{h}_k \in \mathbb{C}^{NN_rL \times 1}, k = \{0, 1\}$ be defined as $\mathbf{h}_k \triangleq \mathbb{H}\text{vec}(\mathbf{P}_k)$, where $\mathbb{H} = \mathbf{I}_L \otimes \mathbf{H} \in \mathbb{C}^{NN_rL \times N_tL}$. The operator vec$(\mathbf{P}_k) \in \mathbb{C}^{N_tL \times 1}$ does the vectorization operation of matrix $\mathbf{P}_k \in \mathbb{C}^{N_t \times L}$. The vectored MIMO system model at the fusion center corresponding to L beacon symbols of N secondary users in (1) can be equivalently written as,

$$\mathbf{y} = (\mathbf{I}_L \otimes \mathbf{H})\text{vec}(\mathbf{P}_k) + \mathbf{w} = \mathbf{h}_k + \mathbf{w} \tag{2}$$

where the noise vector $\mathbf{w} \in \mathbb{C}^{NN_rL \times 1}$ is defined as $\mathbf{w} = \text{vec}(\mathbf{W})$. The multiple beacon signal based cooperative spectrum sensing problem at the fusion center corresponding to the NN_rL received antenna signal $\mathbf{y}$ can be recast as binary hypothesis testing problem, written as,

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{y} = \mathbf{h}_0 + \mathbf{w} \\ \mathcal{H}_1 &: \mathbf{y} = \mathbf{h}_1 + \mathbf{w}, \end{aligned} \tag{3}$$

where the null hypothesis $\mathcal{H}_0$ corresponds to the absence of the primary user signal whereas the alternative hypothesis $\mathcal{H}_1$ corresponds to the presence of primary user signal. The optimal detection rule for the binary hypotheses testing problem in (3) for the AWGN scenario in (2) is the standard matched filter detector [17], i.e. decide alternative hypothesis $\mathcal{H}_1$ when $\mathbf{z}^H\mathbf{h}_1 > 0$, otherwise, $\mathbf{z}^H\mathbf{h}_0 \leq 0$ corresponds to the null hypothesis $\mathcal{H}_0$. Hence, the optimal decision vector $\mathbf{z}$, that minimizes the probability of error, is the perpendicular bisector of the points corresponding to the two competing hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$. Therefore, the optimal decision rule for the multiple beacon based binary hypothesis testing problem in (3) for MIMO cognitive radio networks is obtained by solving the optimization problem, given as,

$$\begin{aligned} \min. \quad & \|\mathbf{z}\|_2 \\ \text{s.t.} \quad & \mathbf{z}^H\mathbf{h}_1 + c \geq 1 \\ & \mathbf{z}^H\mathbf{h}_0 + c \leq -1 \end{aligned} \tag{4}$$

where $\|\cdot\|_2$ denotes $L2-$norm and c is a constant. The above optimization problem can be easily seen to be a convex optimization problem as it has a convex cost function with affine inequality constraints. The closed form solution obtained for the above optimization problem in (4) is same as the one obtained from matched filter. It can be observed that the obtained decision rule is inherently CSI dependent.

A. CSI uncertainty

As it is known from our previous discussion in the introduction section that obtaining accurate CSI is extremely difficult due to the limited feedback and other channel impairments. Hence under such condition only a nominal estimate of the true channel coefficient vector $\mathbf{h}$ can be obtained. Therefore, we model the uncertainty in the obtained CSI estimates as the standard ellipsoidal uncertainty set, similar to the work

[18]. The true channel coefficient vector $\mathbf{h} \in \mathbb{C}^{NN_rL\times 1}$ can be written as,

$$\mathbf{h} = \hat{\mathbf{h}} + \mathbf{S}\mathbf{u}, \tag{5}$$

centered at the estimate of the channel coefficient vector $\hat{\mathbf{h}}$ corresponding to the estimated channel matrix $\hat{\mathbb{H}} \in \mathbb{C}^{NN_rL\times N_tL}$, defined earlier as $\hat{\mathbf{h}} = \hat{\mathbb{H}}\text{vec}(\mathbf{P})$, and vector $\mathbf{u} \in \mathbb{C}^{NN_rL\times 1}$ with $\|\mathbf{u}\| \le 1$. The matrix $\mathbf{S} \in \mathbb{C}^{NN_rL\times NN_rL}$, defined as $\mathbf{S} \triangleq \mathbf{I}_L \otimes \mathbf{S}_L$, with $\mathbf{S}_L \in \mathbb{C}^{NN_r\times NN_r}$ denoting the statistical uncertainty in the true channel coefficient vector of N secondary users corresponding to each beacon vector. Next, we present the framework to obtain multiple beacon based detectors for cooperative spectrum sensing in MIMO cognitive radio networks that are robust against the CSI uncertainty.

III. Multiple Beacon Based Robust Detectors

A. Robust Detector

Using (5) in the inequality constrains of the optimization framework in (4) we can recast the above optimization problem of multiple beacon vector based primary user detection in MIMO cognitive radio networks as,

$$\begin{aligned} \min.\ & \|\mathbf{z}\|_2 \\ \text{s.t.}\ & \mathbf{z}^H(\hat{\mathbf{h}}_0 + \mathbf{S}\mathbf{u}) + c \le -1 \qquad (6) \\ & \mathbf{z}^H(\hat{\mathbf{h}}_1 + \mathbf{S}\mathbf{u}) + c \ge 1. \qquad (7) \end{aligned}$$

The optimal decision rule is the one that maximally separates the ellipsoidal uncertainty sets corresponding to the two hypotheses. One can further simplify the inequality (6) of the above optimization framework as,

$$\begin{aligned} \mathbf{z}^H(\hat{\mathbf{h}}_0 + \mathbf{S}\mathbf{u}) + c &\le -1 \\ \max_{\|\mathbf{u}\|\le 1}. \Big(\mathbf{z}^H(\hat{\mathbf{h}}_0 + \mathbf{S}\mathbf{u}) + c\Big) &\le -1 \qquad (8) \\ \mathbf{z}^H\hat{\mathbf{h}}_0 + \max_{\|\mathbf{u}\|\le 1}.\ \mathbf{z}^H\mathbf{S}\mathbf{u} + c &\le -1 \\ \mathbf{z}^H\hat{\mathbf{h}}_0 + \mathbf{z}^H\mathbf{S}\Big(\frac{\mathbf{S}^H\mathbf{z}}{\|\mathbf{S}^H\mathbf{z}\|}\Big) + c &\le -1 \qquad (9) \\ \mathbf{z}^H\hat{\mathbf{h}}_0 + \|\mathbf{S}^H\mathbf{z}\| + c &\le -1, \qquad (10) \end{aligned}$$

where the left hand side of the inequality in (8) denotes the worst case ellipsoidal distance and (9) follows from the maximization of $\mathbf{z}^H\mathbf{S}\mathbf{u}$ for $\|\mathbf{u}\| \le 1$ occurs for $\mathbf{u} = \frac{\mathbf{S}^H\mathbf{z}}{\|\mathbf{S}^H\mathbf{z}\|}$. Similarly, the second inequality (7) of the optimization framework can be further simplified as,

$$\begin{aligned} \mathbf{z}^H(\hat{\mathbf{h}}_1 + \mathbf{S}\mathbf{u}) + c &\ge 1 \\ \min_{\|\mathbf{u}\|\le 1}. \Big(\mathbf{z}^H(\hat{\mathbf{h}}_1 + \mathbf{S}\mathbf{u}) + c\Big) &\ge 1 \\ \mathbf{z}^H\hat{\mathbf{h}}_1 + \min_{\|\mathbf{u}\|\le 1}.\ \mathbf{z}^H\mathbf{S}\mathbf{u} + c &\ge 1 \\ \mathbf{z}^H\hat{\mathbf{h}}_1 - \|\mathbf{S}^H\mathbf{z}\| + c &\ge 1. \qquad (11) \end{aligned}$$

Hence, using the robust inequalities obtained in (10) and (11), the optimal separating hyperplane that maximizes the worst case separation between the ellipsoidal uncertainty sets corresponding to the two hypotheses can be recast as,

$$\begin{aligned} \min.\ & \|\mathbf{z}\|_2 \\ \text{s.t.}\ & \mathbf{z}^H\hat{\mathbf{h}}_0 + \|\mathbf{S}^H\mathbf{z}\| + c \le -1 \\ & \mathbf{z}^H\hat{\mathbf{h}}_1 - \|\mathbf{S}^H\mathbf{z}\| + c \ge 1. \qquad (12) \end{aligned}$$

The solution to the above cost function yields the multiple beacon based robust decision rule for the problem of primary user detection in MIMO cognitive radio networks. It can be observed that the optimization problem in (12) is a second order cone program (SOCP) optimization problem, hence convex, and can be easily solved using conic solvers. Depending on the signaling scheme employed by the primary user base-station, the set of L vectors broadcasted, i.e., $\mathbf{u}_i(l) \in \mathbb{C}^{N_t\times 1}, 1 \le l \le L$ and $\mathbf{U} = [\mathbf{u}(1), \mathbf{u}(2), \cdots, \mathbf{u}(L)] \in \mathbb{C}^{N_t\times L}$, with $\mathbf{U} \in \{\mathbf{P}_0, \mathbf{P}_1\}$ corresponding to its local decision can either be antipodal or on-off in nature.

1) Antipodal Signaling: Consider antipodal signaling where the primary user base-station broadcasts $\mathbf{U} = -\mathbf{P}_1$ and $\mathbf{U} = \mathbf{P}_1$ respectively indicating the absence and presence of the primary user signal. As defined in (2), $\hat{\mathbf{h}}_1 = \hat{\mathbb{H}}\text{vec}(\mathbf{P}_1)$, the estimate vector $\hat{\mathbf{h}}_1$ corresponding to the null hypothesis $\mathcal{H}_0$ and the alternative hypothesis $\mathcal{H}_1$ for the antipodal signaling schemes reduces to $-\hat{\mathbf{h}}_1$ and $\hat{\mathbf{h}}_1$, respectively. Using the estimates $-\hat{\mathbf{h}}_1$ and $\hat{\mathbf{h}}_1$ for antipodal signaling scheme, the optimization framework in (12) for cooperative spectrum sensing in MIMO cognitive radio networks can be equivalently formulated as,

$$\begin{aligned} \min.\ & \|\mathbf{z}\|_2 \\ \text{s.t.}\ & \mathbf{z}^H\hat{\mathbf{h}}_1 - \|\mathbf{S}^H\mathbf{z}\| \ge 1. \qquad (13) \end{aligned}$$

The optimal decision hyperplane that separates the two hypothesis ellipsoid in (12) for the antipodal signaling is homogeneous, i.e, the constant $c = 0$. To determine the optimum separating hyperplane, the above SOCP optimization problem in (13) can be computed by solving the lagrangian cost function $L(\mathbf{z}, \gamma)$ as,

$$\begin{aligned} L(\mathbf{z},\gamma) &= \|\mathbf{z}\|^2 + \gamma\Big(\|\mathbf{S}^H\mathbf{z}\|^2 - (\mathbf{z}^H\hat{\mathbf{h}}_1 - 1)\Big) \\ &= \mathbf{z}^H\big(\mathbf{I} + \gamma(\mathbf{S}\mathbf{S}^H - \hat{\mathbf{h}}_1\hat{\mathbf{h}}_1^H)\big)\mathbf{z} + 2\gamma\mathbf{z}^H\hat{\mathbf{h}}_1 - \gamma, \end{aligned}$$

where $\mathbf{I}$ is the identity matrix. The optimal $\mathbf{z}_{\text{opt}}$ corresponding to the Lagrange multiplier γ_{opt} can be obtained as,

$$\mathbf{z}_{\text{opt}} = -\gamma_{\text{opt}}(\mathbf{I} + \gamma_{\text{opt}}(\mathbf{S}\mathbf{S}^H - \hat{\mathbf{h}}_1\hat{\mathbf{h}}_1^H))^{-1}, \tag{14}$$

where γ_{opt} are the zeros of the secular equation [18], given as,

$$f(\gamma) = \gamma^2\sum_{i=1}^{NN_rL}\frac{\hat{h}_i^2 q_i}{(1+\gamma q_i)^2} - 2\gamma\sum_{i=1}^{NN_rL}\frac{\hat{h}_i}{(1+\gamma q_i)} - 1,$$

where $\hat{\mathbf{h}}_1 = [\hat{h}_1, \ldots, \hat{h}_i, \ldots, \hat{h}_{NN_rL}]^T$ and $q_i \in \mathbb{R}^n$ are the diagonal elements of the matrix $\mathbf{Q} = \mathbf{S}\mathbf{S}^H - \hat{\mathbf{h}}_1\hat{\mathbf{h}}_1^H$.

2) On-Off Signaling: Often the beacon signaling scheme is non-antipodal in nature, i.e., the beacon matrix $\mathbf{U} = \mathbf{P}$ and $\mathbf{U} = \mathbf{0}_{N_t\times L}$ indicating the presence and absence of the primary signal. Hence, the vector $\hat{\mathbf{h}} = \hat{\mathbb{H}}\text{vec}(\mathbf{P})$ corresponding to the two hypotheses for the non-antipodal signaling scheme reduces to $\mathbf{h}$ and $\mathbf{0}_{NN_rL\times 1}$, respectively. Therefore, the optimal separating hyperplane for the multiple beacon based non-antipodal signaling scenario in MIMO cognitive radio networks can be obtained by the solution of the optimization problem, given as,

$$\begin{aligned} \min.\ & \|\mathbf{z}\|_2 \\ \text{s.t.}\ & \mathbf{z}^H\hat{\mathbf{h}} - \|\mathbf{S}^H\mathbf{z}\| + c \ge 1. \qquad (15) \end{aligned}$$

The closed form expression for the above optimization problem in (15) can be found using a procedure similar to the one obtained in section III-A1.

B. Allied Detectors

In low signal-to-noise (SNR) ratio scenarios, when the two hypotheses ellipsoids corresponding to the two hypothesis may coincide, i.e., the two hypothesis regions can't be separated by a decision hyperplane. We present an optimization framework that maximally relax the inequality constraints in order to obtain a decision hyperplane, from (12), as,

$$\begin{aligned} \min . \quad & d \\ \text{s.t.} \quad & \mathbf{z}^H\hat{\mathbf{h}}_0 + \|\mathbf{S}^H\mathbf{z}\| + c \leq -1 + d \\ & \mathbf{z}^H\hat{\mathbf{h}}_1 - \|\mathbf{S}^H\mathbf{z}\| + c \geq 1 - d \\ & d \geq 0, \end{aligned} \tag{16}$$

where d is the slack variable denoting the measure of constraint relaxation. The decision hyperplane corresponding to the two cost functions of the optimization frameworks in (12) and (16), the perceptible detector would be the one that allows a tradeoff between the two cost functions, i.e., tradeoff detector, obtained as,

$$\begin{aligned} \min . \quad & \|\mathbf{z}\|_2 + \mu d \\ \text{s.t.} \quad & \mathbf{z}^H\hat{\mathbf{h}}_0 + \|\mathbf{S}^H\mathbf{z}\| + c \leq -1 + d \\ & \mathbf{z}^H\hat{\mathbf{h}}_1 - \|\mathbf{S}^H\mathbf{z}\| + c \geq 1 - d \\ & d \geq 0, \end{aligned} \tag{17}$$

where μ is a weighting constant that allows weighting for the above multicriterion objective function.

1) Antipodal Signaling: The antipodal signaling, similar to the one considered in section III-A1, relaxed robust detector for cooperative spectrum sensing in MIMO cognitive radio networks can be equivalently obtained from (13) as,

$$\begin{aligned} \min . \quad & d \\ \text{s.t.} \quad & \mathbf{z}^H\hat{\mathbf{h}}_1 - \|\mathbf{S}^H\mathbf{z}\| \geq 1 - d \\ & d \geq 0, \end{aligned} \tag{18}$$

and antipodal signaling based tradeoff detector be obtained as,

$$\begin{aligned} \min . \quad & \|\mathbf{z}\|_2 + \mu d \\ \text{s.t.} \quad & \mathbf{z}^H\hat{\mathbf{h}}_1 - \|\mathbf{S}^H\mathbf{z}\| \geq 1 - d \\ & d \geq 0. \end{aligned} \tag{19}$$

2) On-Off Signaling: Multiple beacon based relaxed robust detector for the on-off signaling scheme, considered in section III-A2, can be obtained from (15) as,

$$\begin{aligned} \min . \quad & d \\ \text{s.t.} \quad & \mathbf{z}^H\hat{\mathbf{h}} - \|\mathbf{S}^H\mathbf{z}\| + c \geq 1 - d \\ & d \geq 0. \end{aligned} \tag{20}$$

Similarly, multiple beacon based tradeoff detector reduces to,

$$\begin{aligned} \min . \quad & \|\mathbf{z}\|_2 + \mu d \\ \text{s.t.} \quad & \mathbf{z}^H\hat{\mathbf{h}} - \|\mathbf{S}^H\mathbf{z}\| + c \geq 1 - d \\ & d \geq 0. \end{aligned} \tag{21}$$

	Generalised framework	Antipodal Signaling	On-Off Signaling
Robust detector	(12)	(13)	(15)
Relaxed detector	(16)	(18)	(20)
Tradeoff detector	(17)	(19)	(21)

TABLE I
OPTIMIZATION FRAMEWORK EQUATION NUMBERS FOR THE PROPOSED MULTIPLE BEACON BASED CSI UNCERTAINTY AWARE DETECTORS.

The proposed antipodal and on-off signaling based robust detectors are summarized in Table I. Next section presents the simulations for the multiple beacon based robust detectors.

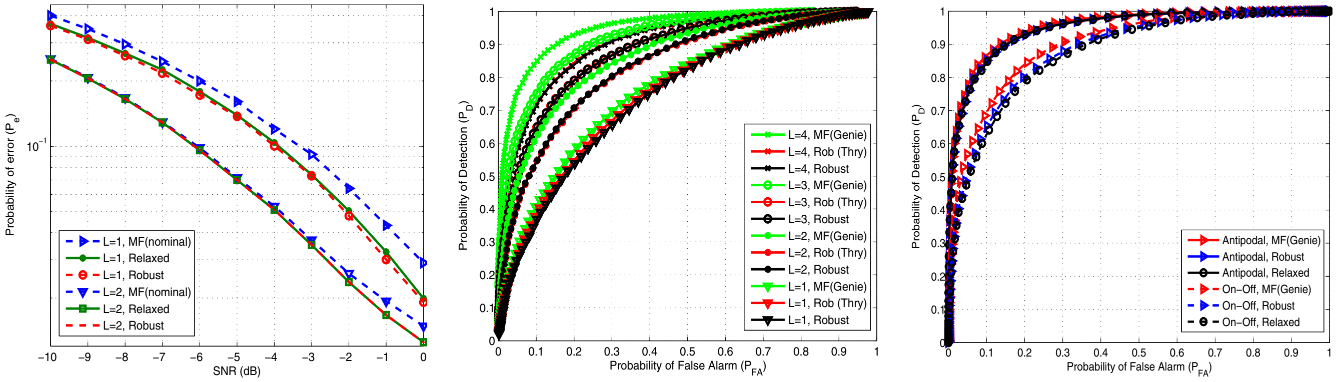
IV. SIMULATION RESULTS

We consider a cognitive radio network with $N = 2$ secondary users each having $N_r = 2$ receive antennas and a primary user base station having $N_t = 2$ transmit antennas that broadcast orthogonal beacon matrix $\mathbf{P} \in \mathbb{C}^{2\times L}$ with $\mathbf{P}\mathbf{P}^H = L\mathbf{I}_2$ for varying number of beacon vectors $L = \{1, 2, 3, 4\}$. Hence, for the scenario of on-off beacon signal matrices we consider $\mathbf{P}_0 = \mathbf{0}_{2\times L}$ that corresponds to the absence of the primary signal and $\mathbf{P}_1 = \sqrt{2}\mathbf{P} \in \mathbb{C}^{2\times L}$ corresponds to the presence of the primary signal. Similarly, for the antipodal beacon signal matrices $\mathbf{P}_0 = -\frac{1}{\sqrt{2}}\mathbf{P}$ and $\mathbf{P}_1 = \frac{1}{\sqrt{2}}\mathbf{P} \in \mathbb{C}^{2\times L}$ corresponds to the absence and presence of the primary signal. Similar to [16], we incorporate different levels of CSI uncertainty. This is characterized by defining the statistical variation matrix $\mathbf{S} = \mathbf{I}_L \otimes \mathbf{S}_L$ such that $\mathbf{S}_L = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$, where $\mathbf{\Lambda} = \mathcal{D}(\boldsymbol{\lambda})$ is a diagonal matrix with principle diagonal as the elements of the vector $\boldsymbol{\lambda} = [1.6, 1.4, 1.2, 1]^T$ of order $NNr\times 1$ and $\mathbf{U}$ is a random unitary matrix of order $NNr \times NN_r$.

Fig. 1(a) plots the probability of error vs. signal-to-noise (SNR) ratio to characterize the detection performance of the proposed robust detector (Robust) in (13), relaxed detector (Relaxed) in (18) with the nominal CSI based matched filter detector (MF Nominal). It can be readily observed that the detection performance of the proposed robust detectors significantly outperforms the uncertainty agnostic matched filter detector (MF Nominal). Further the detection performance of the proposed detectors improves by increasing the number of beacon vectors L.

Next, in Fig. 1(b) we plot the probability of false alarm (P_{FA}) vs. probability of detection (P_D) to obtain the receiver operating characteristics (ROC) of the proposed robust detector (Robust) in (13) and the closed form expression obtained in (14) corresponding to the robust detector (Theory) along with the genie aided true channel coefficient based matched filter detector (MF Genie) at SNR$= -15$ dB for increasing number of beacon vectors $L = \{1, 2, 3, 4\}$. It can be observed that the detection performance of the proposed multiple beacon vector based detector improves with the increase in the number of beacon vectors. It is also observed that the detection performances obtained via simulation exactly matches the detection performances obtained via the closed form solution in section III-A1.

Note, the average power corresponding to the antipodal and on-off signaling schemes in Fig. 1(a) and Fig. 1(b) are kept unequal. These values are chosen such that they yield equal deflection coefficient values [17] for the two signaling scenarios. Hence, the detection performance of the antipodal



(a) (b) (c)

Fig. 1. (a) Probability of error vs. SNR comparison between the nominal estimate based matched filter (MF Nominal), robust detector (Robust) and relaxed robust detector (Relaxed) with $N_r = 2$, $N_t = 2$, $N = 2$ for varying number of beacon vectors $L \in \{1, 2\}$. (b) Probability of false alarm vs. probability of detection comparison between the true channel coefficient based genie aided matched filter (MF Genie), robust detector (Robust) and robust detector obtained in section III-A1 (Rob Thry) with $N_r = 2$, $N_t = 2$, $N = 2$, SNR $= -15$ dB for varying number of beacon vectors $L \in \{1, 2, 3, 4\}$. (c) Probability of false alarm vs. probability of detection comparison between the robust detector (Robust), relaxed robust detector (Relaxed) and genie aided matched filter (MF Genie) with antipodal signaling $\{+\mathbf{P}, -\mathbf{P}\}$ and on-off signaling $\{\sqrt{2}\mathbf{P}, \mathbf{0}\}$ for $N_r = 2$, $N_t = 2$, $N = 2$, $L = 1$ and SNR $= -10$ dB.

signaling based detectors, namely the robust detector in (13), relaxed detector in (18) and tradeoff detector in (19), have similar detection performance from their on-off signaling counterparts, namely the robust detector in (15), relaxed detector in (20) and tradeoff detector in (21), respectively.

Therefore, Fig. 1(c) presents the simulation comparison between the set of detectors obtained for the antipodal signaling, i.e., $\mathbf{U} \in \{+\mathbf{P}, -\mathbf{P}\}$, and the on-off signaling, i.e., $\mathbf{U} \in \{\sqrt{2}\mathbf{P}, \mathbf{0}\}$, corresponding to the two hypotheses under equal average power. The detection performance of the proposed antipodal signaling based set of detectors have superior detection performance over the on-off signaling based set of detectors, respectively. This necessarily happens due to the better deflection coefficient value of the antipodal signaling schemes in comparison to the on-off signaling schemes.

V. Conclusion

We presented multiple beacon vectors based novel detectors for robust spectrum sensing in MIMO cognitive radio networks under CSI uncertainty. We employed an optimal linear discriminant framework to develop robust detectors for the problem of primary user detection in MIMO cognitive radio networks that incorporated CSI uncertainty. We obtained signaling specific robust detectors, namely the antipodal signaling based robust detector and the on-off signaling based robust detector. Also, we obtained the closed form expressions for the proposed optimization problems. Further, we presented the allied detectors that are advantageous under low SNR conditions. The simulation results demonstrated a superior detection performance of the proposed multiple beacon signaling based robust detection schemes over uncertainty agnostic schemes for cooperative spectrum sensing in MIMO cognitive radio networks.

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